The Budget Constraint

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Introduction

- **Basic Idea of Consumer Theory**: Consumers choose the **BEST** bundle of goods that they can **AFFORD**
- We will first focus on what we mean by "afford"
- To identify affordability we need to know:
  - What goods a person is buying
  - The price of each good
  - The person's income

Notation

- x_1: An amount of good 1
  - Subscript denotes the good (1, 2, c, t, etc.)
  - Variable denotes the amount of the good (x, y, z, etc.)
  - e.g. If a good is cups of coffee, then $x_c = 3$ and $y_c = 5$ represent 3 and 5 cups of coffee respectively
- p_1: Price of good 1
  - Subscript denotes the good (1, 2, c, t, etc.)
  - p denotes the price of the good
  - e.g. If tea costs \$2 a cup, then $p_t = 2$
- m: The amount of money a person has to spend (Income)
  - e.g. If a person has \$100 to spend, then $m = 100$

Definitions

- **Consumption Bundle (x_1, x_2)**: A list of two numbers that indicate how much of good 1 and good 2 a person chooses to consume
  - Sometimes denoted by a capital letter
    - $X = (x_1, x_2)$ or $A = (y_1, y_2)$
  - e.g. If good 1 is coffee and good 2 is tea, then (3,5) represents a bundle of 3 coffees and 5 teas
- **Budget Constraint**: Equation of all possible combinations of consumption bundles a person can **afford**, given prices and income
  - $p_1 x_1 + p_2 x_2 \leq m$
  - IOW: The total cost of the goods cannot exceed your income
  - Hence we are **constrained** by our **budget**
  - e.g. If you have \$20, $p_1 = 2$, and $p_2 = 5$, then your budget constraint is $2x_1 + 5x_2 \leq 20$

Definitions

- **Budget Set**: Set of all **affordable** consumption bundles, given prices and income
  - e.g. Budget Constraint: $2x_1 + 5x_2 \leq 20$
  - $A = (1, 1)$, $B = (5, 0)$, $C = (0, 4)$, $D = (5, 2)$ are *some* consumption bundles in this budget set
  - $E = (5, 3)$, $F = (0, 5)$ are not bundles in this set; we can't afford them
- **Budget Line**: The set of bundles that exhaust the income, given prices
  - $p_1x_1 + p_2x_2 = m$
  - e.g. Bundles C and D are on the budget line, but A and B are not

Example: Graphing the Budget Constraint

- Coffee (x_1) costs \$2, Tea (x_2) costs \$5, and our income is \$20
- Budget Line: $2x_1 + 5x_2 = 20$
- Let's find the intercepts:
  - Let $x_2 = 0$

    $2x_1 + 5(0) = 20$

    $2x_1 = 20$ (Simplify)

    $x_1 = 10$ (Divide by 2)

    One point is A=(10,0)
  - Let $x_1 = 0$

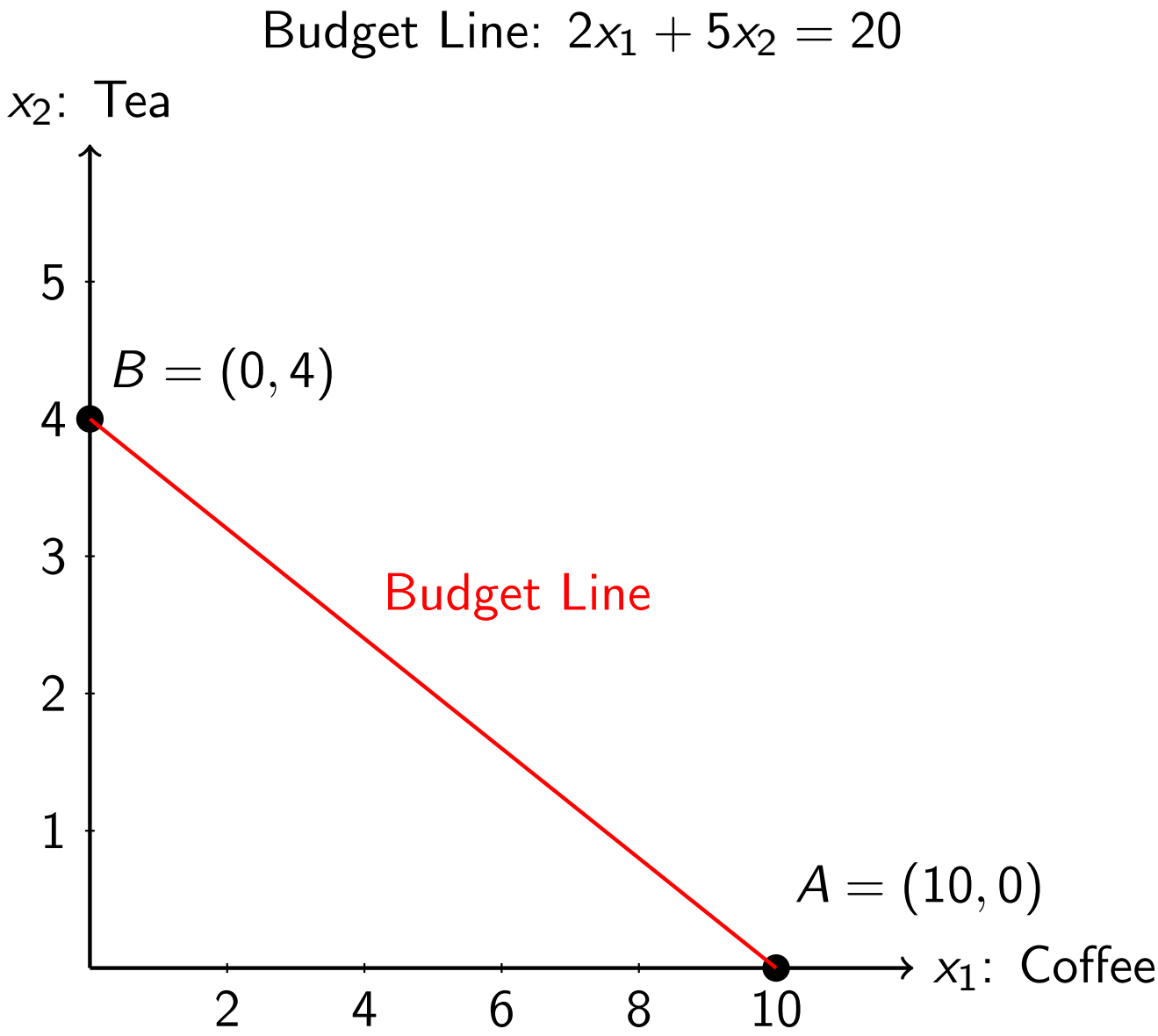
    $2(0) + 5x_2 = 20$

    $5x_2 = 20$ (Simplify)

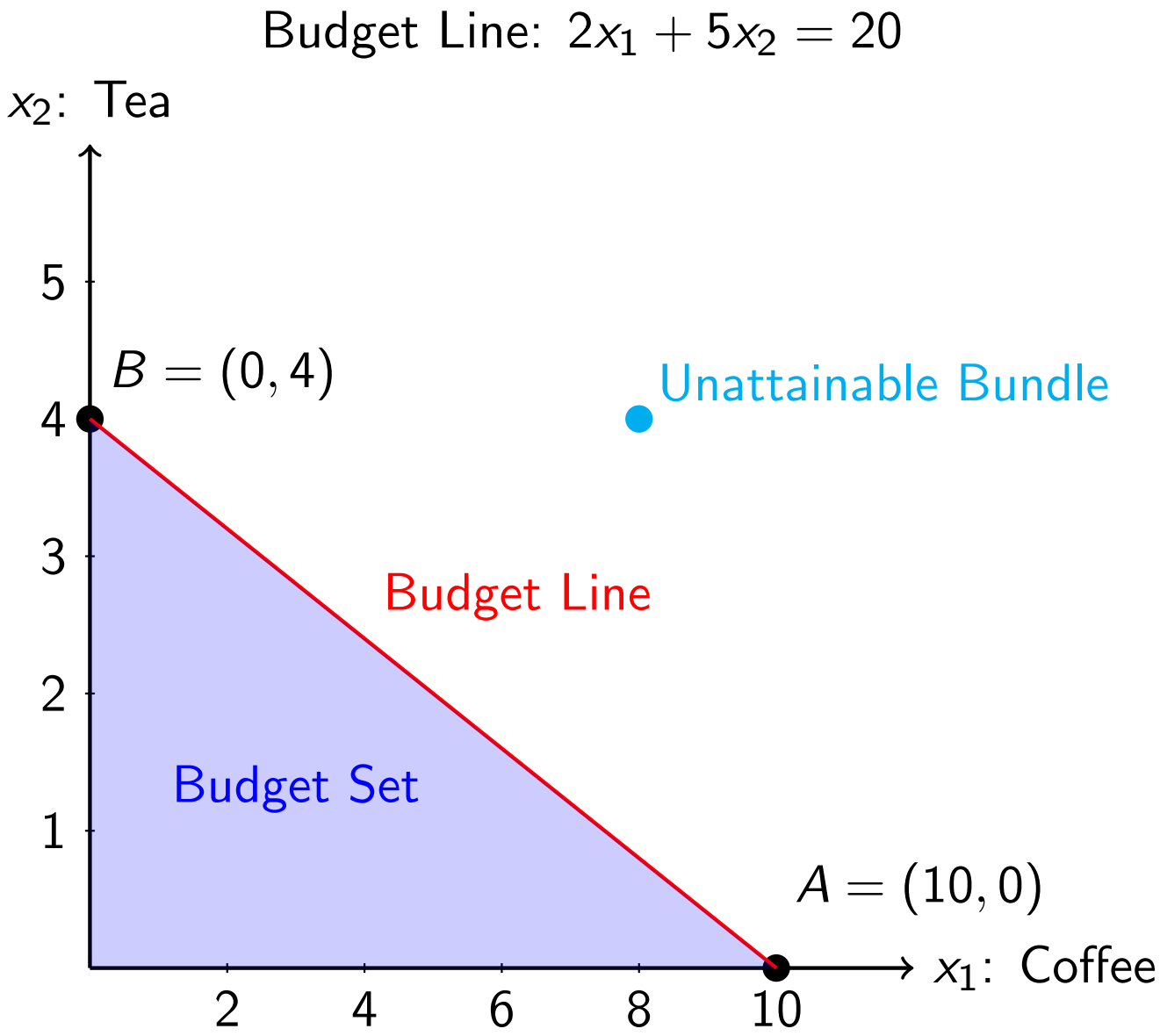
    $x_2 = 4$ (Divide by 5)

    Another point is B=(0,4)
- We have two points, so we can plot and connect them

Example: Graphing the Budget Constraint



Example: Graphing the Budget Constraint



How To Graph The Budget Line

1. Write down the budget Line
   - $p_1 x_1 + p_2 x_2 = m$
2. Plug in p_1, p_2, and m
3. Solve for the intercepts
   - Plug in $x_1 = 0$ and solve for x_2
   - Repeat with $x_2 = 0$
4. Plot the two intercepts: $(0, x_2)$, $(x_1, 0)$
5. Connect the two points

- All the points on and below the budget line represent the **Budget Set**
- All the points above the budget line are unattainable

Opportunity Cost

- **Opportunity Cost**: How much of a good a consumer will give up, to get one more unit of the other good
  - It is negative because the **opportunity** to consume more of a good will **cost** you some of the other good
- Budget Line: $2x_1 + 5x_2 = 20$
  - Let's solve for x_2

    $5x_2 = 20 - 2x_1$ (Subtract $2x_1$ from both sides)

    $x_2 = \frac{20}{5} - \frac{2}{5}x_1$ (Divide both sides by 5)

    $x_2 = -\frac{2}{5}x_1 + 4$ (Rearrange and simplify)
  - The opportunity cost of consuming 1 extra cup of coffee is $\frac{2}{5}$ cups of tea
  - If you instead solved for x_1, the opportunity cost of consuming 1 extra cup of tea would be 2.5 cups of coffee

Opportunity Cost

- Opportunity cost of consuming an extra unit is also just the price ratio of the goods
- Budget Line: $p_1 x_1 + p_2 x_2 = m$
- Let's rearrange this to solve for x_2

  $p_1 x_1 + p_2 x_2 = m$

  $p_2 x_2 = m - p_1 x_1$ (Subtract $p_1 x_1$ from both sides)

  $x_2 = \frac{m}{p_2} - \frac{p_1}{p_2} x_1$ (Divide both sides by p_2)

  $x_2 = -\frac{p_1}{p_2} x_1 + \frac{m}{p_2}$ (Rearrange)
- Opportunity cost of consuming an extra unit of x_1: $-\frac{p_1}{p_2}$
- Opportunity cost of consuming an extra unit of x_2: $-\frac{p_2}{p_1}$

What Affects the Budget Line?

- The affordability of a bundle of goods depends on a consumer's **income** and the **prices** of the goods
- **Income**
  - When income increases (decreases) and prices stay the same, the budget line shifts outward (inward) and the slope remains the same
- **Prices**
  - If p_1 increases (decreases) while holding p_2 and income fixed...
    - x_2 intercept does not change: $(0, x_2)$
    - Budget line/slope becomes steeper (flatter)
    - If it is p_2 that changes, it has the opposite effect
  - p_1 and p_2 both increase (decrease) while income is fixed
    - Budget line shifts inward (outward)
    - Slope depends on the ratio of the changes
- **Income and Price**
  - Depends...

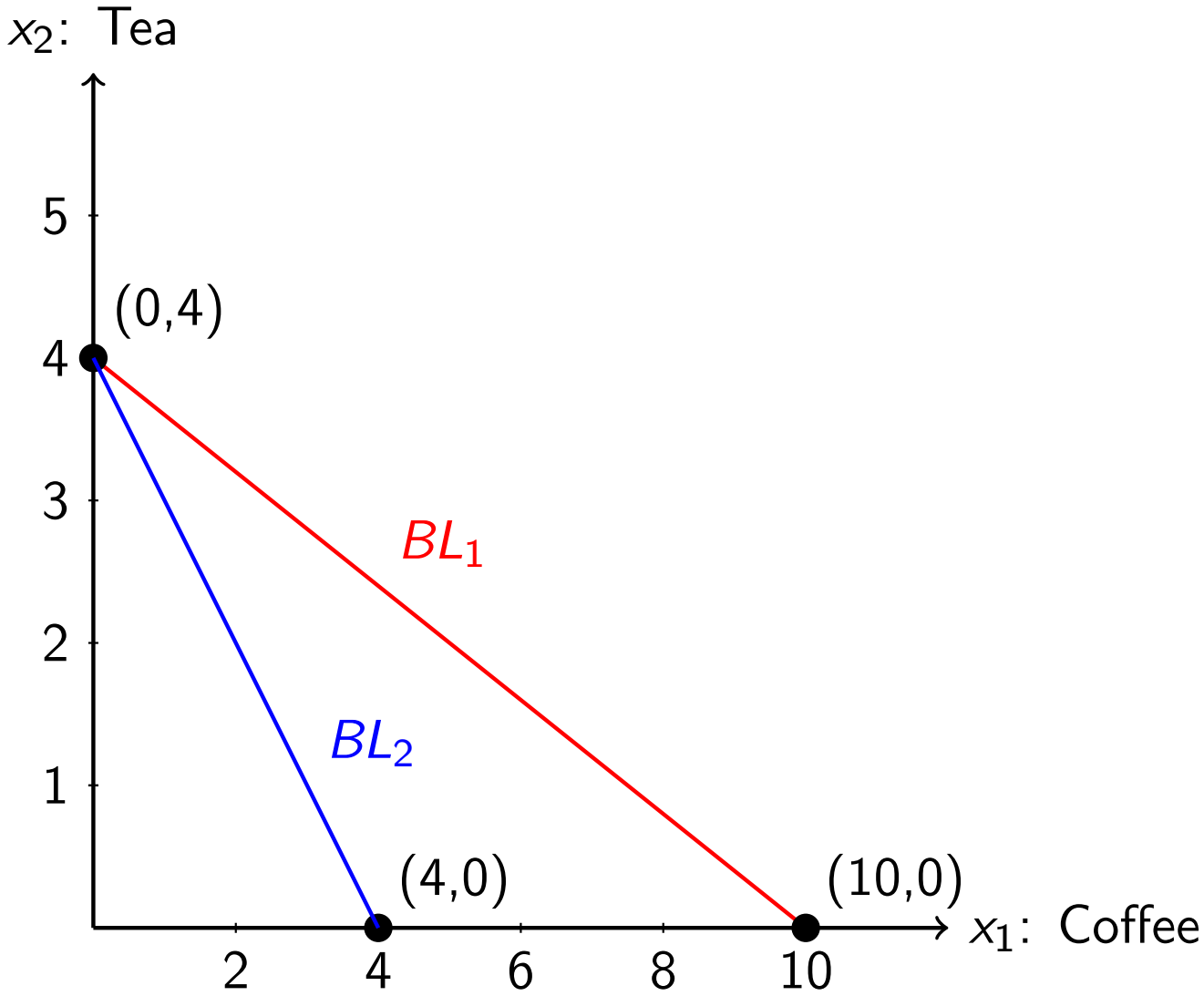
Example: Budget Line Effects

$$p_1 = 2,\ p_2 = 5,\ m = 20$$

- Suppose Brazil experiences a freak frost that damages many coffee groves. Because of this, the price of coffee (p_1) increases to \$5.
- $p_1' = 5,\ p_2 = 5,\ m = 20$
- New Budget Line: $5x_1 + 5x_2 = 20$
- New Intercept(s): $(0, 4)$ and $(4, 0)$

Example: Budget Line Effects

BL_1: $2x_1 + 5x_2 = 20$ BL_2: $5x_1 + 5x_2 = 20$



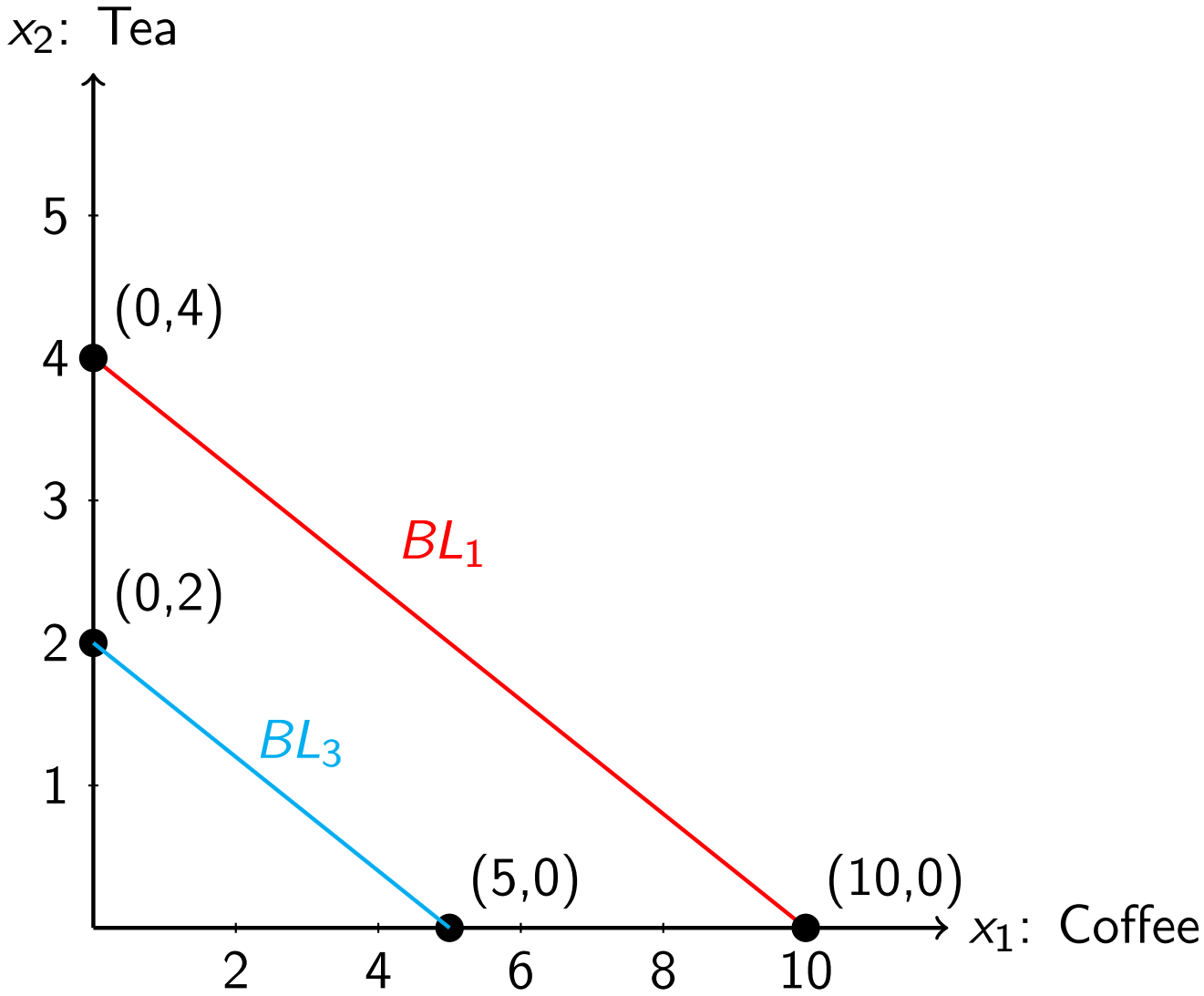
Example: Budget Line Effects

$$p_1 = 2,\ p_2 = 5,\ m = 20$$

- The frost never happened, but you parked in ramp 6 without paying and got a \$10 ticket that comes out of your income
- $p_1 = 2,\ p_2 = 5,\ m' = 10$
- New Budget Line: $2x_1 + 5x_2 = 10$
- New Intercepts: $(0, 2)$ and $(5, 0)$

Example: Budget Line Effects

BL_1: $2x_1 + 5x_2 = 20$ BL_3: $2x_1 + 5x_2 = 10$



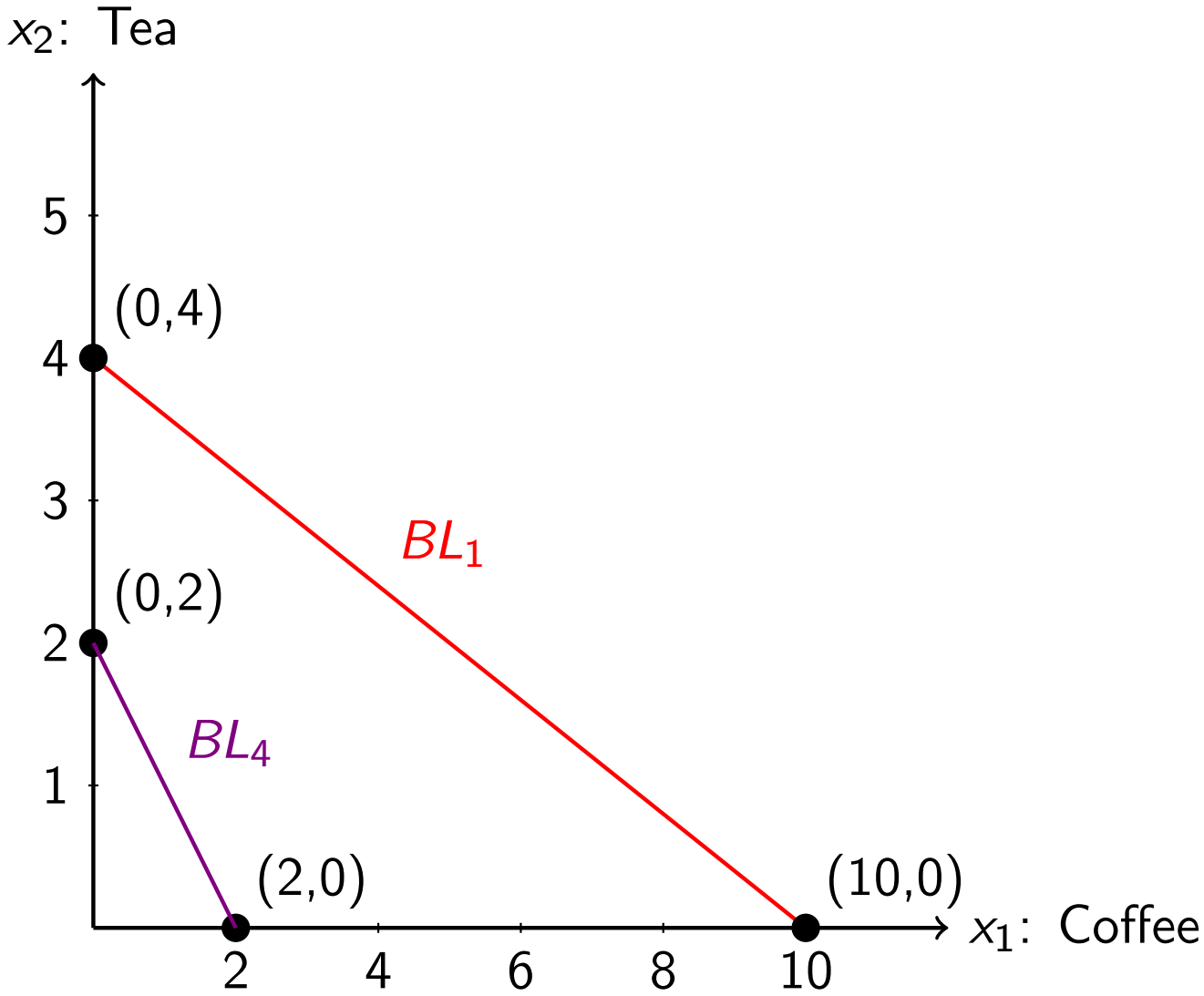
Example: Budget Line Effects

$$p_1 = 2,\ p_2 = 5,\ m = 20$$

- Now it's a really bad day and the frost occurred and you got a ticket
- $p_1' = 5,\ p_2 = 5,\ m' = 10$
- New Budget Line: $5x_1 + 5x_2 = 10$
- New Intercepts: $(0, 2)$ and $(2, 0)$

Example: Budget Line Effects

BL_1: $2x_1 + 5x_2 = 20$ BL_4: $5x_1 + 5x_2 = 10$



Taxes, Subsidies, and Rationing

- Many economic policies that governments employ can affect a consumer's budget constraint
- **Quantity Tax**: Consumers pay a certain amount to the government for each unit of the good purchased
  - Gas tax: 30 cents per gallon in Michigan
  - Same effect on the budget line as raising prices
  - e.g. Suppose gas costs $p_g = 2.50$/gallon. But the government imposes a new tax of \$0.50/gallon. The new price becomes $p'_g = 3.00$/gallon
- **Value (Ad Valorem) Tax:** Consumers pay the government a percentage of the value of the good purchased
  - Sales Tax: 6% in Michigan
  - Same effect on the budget line as raising prices
  - e.g. In Florida, there are sales tax holidays. Suppose a backpack there costs $p_b = 20$ during the holiday. But on a non-holiday, it is subjected to a 6% tax. The new cost is $p'_b = 1.06 p_b = 21.20$

Taxes, Subsidies, and Rationing

- **Quantity Subsidy**: Governments give an amount to consumers based on how many units of a good are purchased
  - Agricultural industry: corn, wheat, soybean, etc
  - Has the same effect on the budget line as decreasing prices
  - e.g. Suppose your internet costs $p_i = 70$/month. But, you now qualify for the ACP and its subsidy of \$30. The new cost is $p_i' = 40$
- **Value (Ad Valorem) Subsidy**: Governments give consumers a percentage of the value of the good purchased
  - ACA, small business, and solar power subsidies
  - Same effect on the budget line as decreasing prices
  - e.g. A solar panel costs $p_s = 100$, but the government offers a subsidy of 20% total cost. The new price becomes $p_s' = .80p_s = 80$
- Taxes and subsidies affect the budget line in the same way, but have opposite effects

Taxes, Subsidies, and Rationing

- **Lump Sum Taxes (Subsidies)**: The government will take (give) a fixed amount of money to a consumer, regardless of purchase behavior.
  - Car registrations (tax) or stimulus checks (subsidy)
  - Same thing as a reduction (increase) in income
  - The budget line will shift inward (outward)
  - e.g. Suppose you have an income of $m = \$30,000$ and the government decides to give out stimulus checks worth \$1,500 to all adults who made less than \$80,000 last year. Then your income becomes $m' = \$31,500$
- Quantity and Value taxes/subsidies tilt the budget line while lump sum shifts it

Taxes, Subsidies, and Rationing

- **Rationing**: Consumption of some good cannot exceed a certain amount
  - e.g. WWII (butter, meat), hurricane season (water, flashlights, gas), Covid (toilet paper, cleaning supplies)
  - Causes a vertical or horizontal cut off on the budget line
- **Kinked Budget Line**: Occurs when the price of a good changes after a certain amount is bought

Example: Rationing

Budget Line: $2x_1 + 5x_2 = 20$

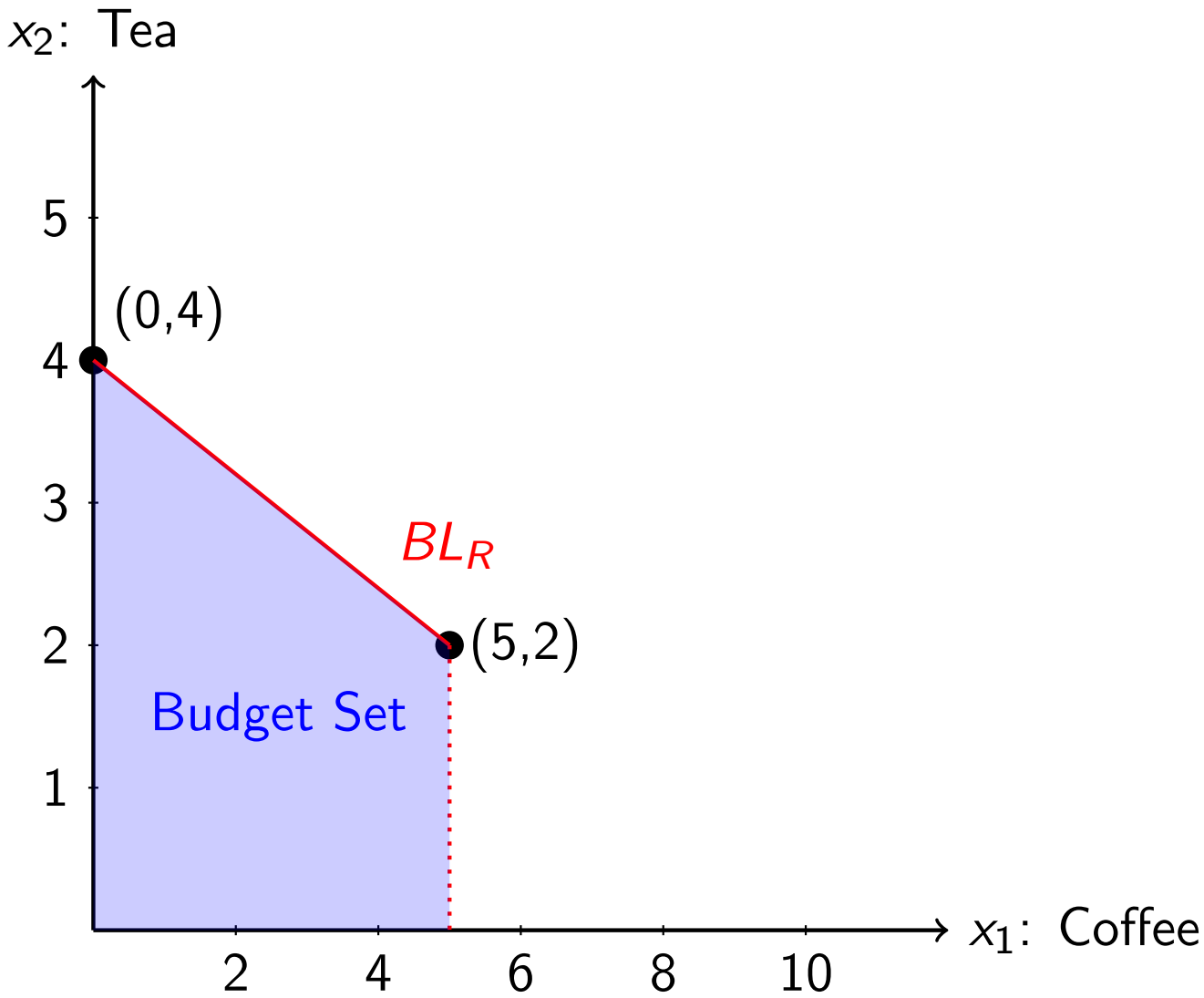
- It's finals week and MSU has convinced all local coffee shops to limit customers to five coffees. All prices and income remain the same
- Before the rationing, the maximum amount of coffees we could buy was 10: (10,0)
- Now the maximum amount of coffees we are allowed to buy is 5
- If we buy 5 coffees, we have enough money left to buy 2 teas

  $2(5) + 5x_2 = 20$

  $x_2 = 2$
- $(5, 2)$ is our new end point
- $(0, 4)$ is still our x_2 intercept

Example: Rationing

BL_R: $2x_1 + 5x_2 = 20$ *Rationed to 5 coffees



How to Graph a Rationed Budget Line

1. Identify which good(s) is being rationed and at what amount
2. Write down the budget line
3. Plug in the maximum amount you are allowed to buy of the rationed good
4. Solve for the remaining variable
   - This amount and the maximum of the rationed good will be your endpoint
5. Now plug in 0 for the rationed good in the budget line and solve for the non-rationed good
   - This will be your intercept
6. Connect these two points
7. Draw a dashed line from the endpoint to the axis that does not have an intercept

Example: Kinked Budget Line

$$2x_1 + 5x_2 = 20$$

- It's finals week, but now instead of rationing coffee, MSU has convinced all local coffee shops to charge an extra \$2 for every coffee bought after the fifth one.
- This change in p_1 applies only to quantities purchased after 5

  $p_1 = 2$ for $0 \leq x_1 \leq 5$

  $p_1' = 4$ for $5 < x_1$
- Kink point:

  $2(5) + 5x_2 = 20$ (Price changes at $x_1 = 5$)

  $x_2 = 2$ (Simplify)

  Kink Point: (5,2)

Example: Kinked Budget Line

- Intercept with original prices:

  $2(0) + 5x_2 = 20$ (Plug 0 in for the good that changes price)

  $x_2 = 4$ (Simplify)

  Intercept: (0,4)

- Intercept with new prices:

  $2 \cdot 5 = 10$ (Total cost of coffee before price change)

  $4x_1 + 5x_2 = 20 - 10$ (subtract 10 from income)

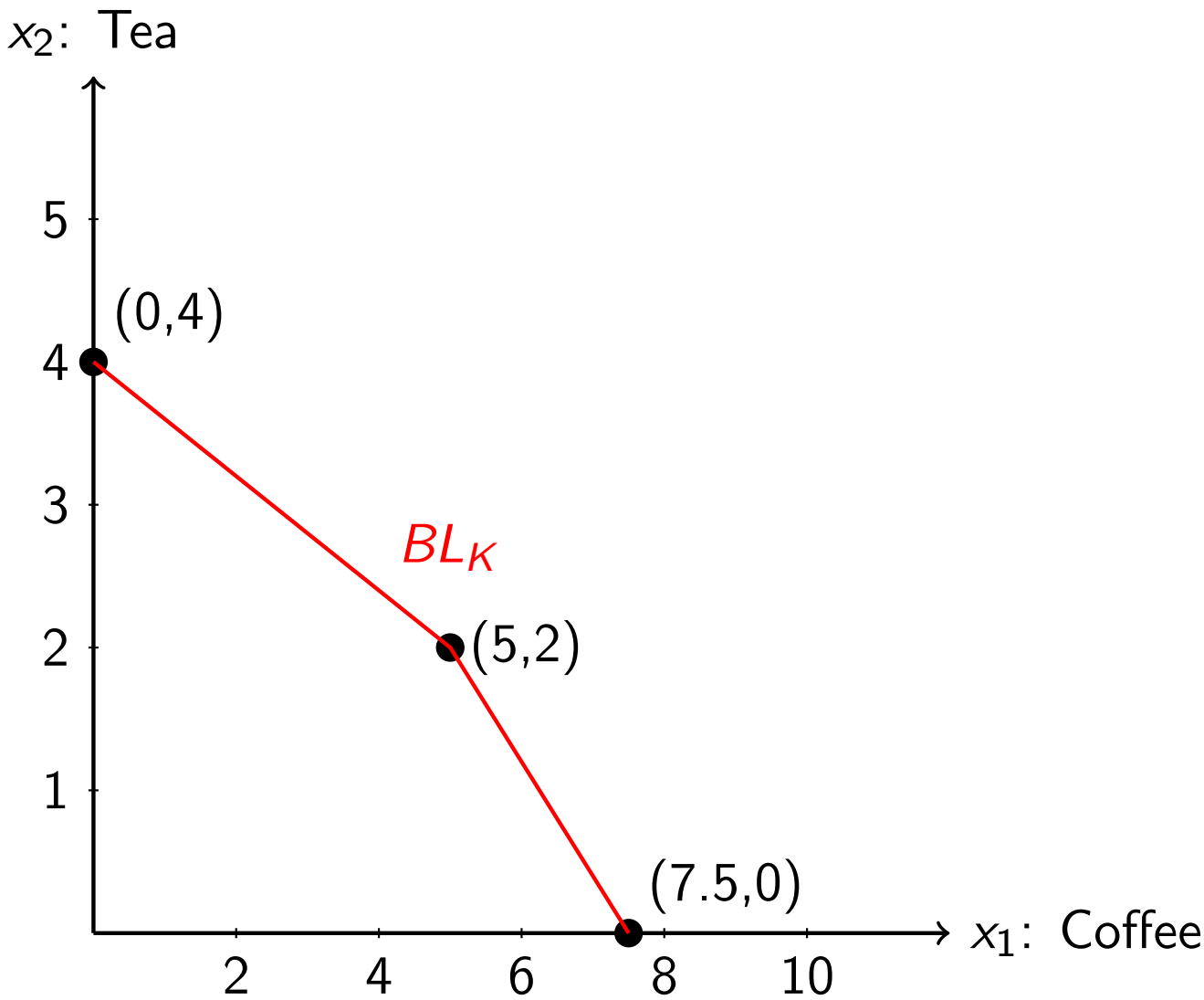
  $4x_1 + 5(0) = 10$ (Plug 0 in for the good that doesn't change price)

  $x_1 = 2.5$ (Simplify)

  $x_1 = 2.5 + 5 = 5.5$ (Add the quantity where it changes)

  Intercept: (7.5,0)

Example: Kinked Budget Line



How to Graph a Kinked Budget Line

1. Identify which good(s) changes price and at what quantity
2. Write down the budget line with the prices before the change
3. Plug in the amount of the good you can buy before it changes price
4. Solve for the good whose price doesn't change
   - This is your kink point
5. Rewrite the budget line with the prices before the change
6. Plug in 0 for the good whose price changes
7. Solve for the good whose prices doesn't change
   - This is one of the intercepts

How to Graph a Kinked Budget Line

8. Rewrite the budget line with the prices after the change
9. Multiply the maximum you can buy of the price changing good before the price changes by the original price of the good
10. Subtract that amount from the income
11. Plug in 0 for the good whose price doesn't change
12. Solve for the good whose price does change
13. Add the amount we could buy before the price change to quantity solved for
    - This is the other intercept

Why Only Two Goods?

- So far, we've only discussed decisions made between two goods
- But that is often unrealistic
- We can think of one good as a specific good and the other as representing all other possible goods
  - **Composite Good**: Everything else a person might consume other than good 1
- This is a special case ONLY for when we are talking about more than two goods
- In this class, we will assume two goods are enough and leave composite goods to advanced classes.